

Fluids and vortex from constrained fluctuations around C-metric black hole

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Abstract

By foliating the four-dimensional C-metric black hole spacetime, we consider a kind of initial-value-like formulation of the vacuum Einstein's equation, the holographic inception data is a double consisting of the induced metric and the Brown-York energy momentum tensor on an arbitrary starter hypersurface. Then by perturbing the inception data that generates the background spacetime, it is shown that, in an appropriate limit, the fluctuation modes are governed by the continuity equation and the Cauchy momentum equation which describe the momentum transport in non-relativistic viscous fluid. Moreover, since the 2-dimensional near horizon hypersurface is always conformally flat, we can map the fluid system onto a flat Newtonian space and thus establishing yet another example of the Gravity/Flat space fluid correspondence found recently in our works [47, 48]. It turns out that the flat space fluid behaves as a pure vortex.

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1 Introduction

In the past two decades the development of fundamental physics has been greatly promoted by the recognition of the holographic principle which emerges in the study of black holes. This bold principle was originally put forward by 't Hooft [1] and Susskind [2], and first realized concretely by Maldacena [3] in the context of string theory. In this implementation a conjectured equivalence was established between the four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills gauge theory and the superstring theory on $\text{AdS}_5 \times \text{S}^5$. Although the origin of the connection is still mysterious, this well-known AdS/CFT correspondence has been applied in many aspects as an efficient tool to perform analytical calculations in strongly coupled systems [4–6]. A canonical application of this correspondence is the analysis of shear viscosity in the strongly coupled SYM theory [4], in which the hydrodynamics arises as a classical description for the behavior of any interacting quantum field theory at long-wavelength and low-frequency. Then in the framework AdS/CFT correspondence a connection between gravity in asymptotically AdS and relativistic hydrodynamics in one less dimension was revealed [7–12], and this is known as the Gravity/Fluid correspondence, and the relativistic gravitational field equations were reduced into the incompressible Navier-Stokes equation in a appropriate scaling limit [13].

Actually, the Gravity/Fluid correspondence can be realized independent of AdS/CFT correspondence, and it's origin could be dated back to the late 1970's, long before the holographic principle was proposed. Here we refer to the membrane paradigm [14–17], in which the local dynamical quantities of black holes was first studied “holographically”, and shown to be governed by the Navier-Stokes equations. Despite the technical differences, the viscosity/entropy ratio from membrane paradigm at horizon and AdS/CFT at spatial infinity was surprisingly the same, which indicates a correlation between these two approaches [18–20]. Under such background the Gravity/Fluid correspondence at arbitrary cutoff was constructed [21]. In it's original version this was achieved by considering linearized Einstein equations while making hydrodynamic expansion and imposing appropriate boundary conditions. To be specific, the scalar and tensor modes of the fluctuations are fixed by the ingoing-wave boundary conditions on the horizon and Dirichlet boundary conditions at the cutoff, and the only dynamical modes are the vector fluctuations governed by the linearized Navier-Stokes equations. Shortly afterwards in [22] the analysis was improved to the nonlinear case, more importantly the hydrodynamic expansions and the boundary conditions are shown to be mathematically equivalent to the near horizon expansions and Petrov-like boundary conditions respectively [23].

In the recent years this reformulated Gravity/Fluid correspondence was greatly generalized [24–46], especially in [47] we studied a type of constrained perturbations around a class of black holes with curved horizons, and in the near horizon region (later this was generalized to finite cutoff [48]), we find that such kind of Petrov-type-fluctuations could be mapped to a forced compressible viscous fluid in flat space of one less dimension. In these works we went beyond the framework of the bulk/boundary

correspondence, and the relaxation of bulk/boundary restriction may possibly be a motivator and reminder of deeper understanding about the holographic principle, also we expect that this further generalization of the Gravity/Fluid correspondence could unearth more potentiality of gravity as a powerful tool in the study of fluid dynamics. But we are far from being optimistic, since we have not yet achieved a complete construction, one of the major obstacles is the unusual body force of the fluid system pertaining to the surface stress. To make sense of the external force we need to apply our analysis to other type of solutions of the Einstein's equation. So, in this paper we take that first step and study a fluid correspondence of a well-known four-dimensional accelerating black hole solution, i.e. the C-metric black hole. It turns out that in this case the form of the external force can be better understood, and more intriguingly, the flat space version of the fluid system behaves like a pure vortex.

2 The C-metric reformulated in an appropriate coordinate system

The whole construction relies on the C-metric solution of the vacuum Einstein equation. The line element reads

$$ds^2 = \frac{1}{\mathcal{A}} \left(-Q dt^2 + \frac{dr^2}{Q} + \frac{r^2}{P} d\theta^2 + Pr^2 \sin^2 \theta d\varphi^2 \right), \quad (1)$$

where

$$\begin{aligned} \mathcal{A}(r, \theta) &= (1 + \alpha r \cos \theta)^2, \\ Q(r) &= (1 - \alpha^2 r^2) \left(1 - \frac{2m}{r} \right), \\ P(\theta) &= 1 + 2\alpha m \cos \theta, \end{aligned}$$

and $0 \leq 2\alpha m < 1$. This solution can be viewed as a one-parameter generalization of the Schwarzschild metric and can be interpreted as two black holes accelerating apart with a constant proper acceleration [49]. The two black holes are separated by an accelerating horizon, so that on each causal patch of the spacetime there is only one black hole with event horizon located at $r = 2m$, and the acceleration horizon is located at $r = \frac{1}{\alpha}$. The two positive parameters m and α respectively correspond to the mass and the proper acceleration of the black holes.

To construct the hydrodynamic equations in isotropic coordinates, it is desirable to introduce a conformal isotropic coordinate system in the angular part of the line element. The new coordinates (x^1, x^2) replace the old ones (θ, φ) via the relations

$$\begin{aligned} x^1 &= w(\theta) \cos \varphi, \\ x^2 &= w(\theta) \sin \varphi, \end{aligned}$$

where

$$w(\theta) = \left(\left(\frac{P}{\sin \theta} \right)^{2\alpha m} \tan \frac{\theta}{2} \right)^{\frac{1}{1-(2\alpha m)^2}}$$

plays the role of a radial coordinate on the (w, φ) “plane” and it is a univariate function in θ ranging from 0 to $+\infty$. Though seemingly weird, the construction of this new coordinate system is straightforward, and we put the details in the Appendix.

Using the above transformation, the line element (1) could be rewritten under the coordinates $x^\mu = (t, r, x^1, x^2)$ in the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{A} \left(-Q dt^2 + \frac{dr^2}{Q} + r^2 e^{\Phi(x^i)} \delta_{ij} dx^i dx^j \right), \quad (2)$$

in which

$$\begin{aligned} A(r, x^i) &= (1 + \alpha r Z)^2, \\ e^{\Phi(x^i)} &= \frac{(1 + 2\alpha m Z)(1 - Z^2)}{\delta_{ij} x^i x^j}, \quad (i = 1, 2) \end{aligned}$$

where

$$Z(x^i) = \cos(\theta(x^i))$$

is an implicit function of x^i . For calculation purposes, it is desirable to rewrite the expression $\delta_{ij} x^i x^j$ in terms of the implicit function $Z(x^i)$,

$$\delta_{ij} x^i x^j = \left(\sqrt{\frac{1-Z}{1+Z}} \left(\frac{(1+2\alpha m Z)^2}{1-Z^2} \right)^{\alpha m} \right)^{\frac{2}{1-(2\alpha m)^2}} \quad (3)$$

because of the simple relation

$$\partial_i Z = -e^\Phi x_i,$$

which can be verified directly. This simple relation will be very facilitating in the following calculations.

3 Constraints on initial hypersurface

We start by foliating the C-metric background by three-dimensional timelike hypersurfaces defined by $r = \text{const.}$ The bulk line element could be expressed as

$$ds^2 = \frac{dr^2}{AQ} + \gamma_{ab} dx^a dx^b = \frac{dr^2}{AQ} + \frac{1}{A} \left(-Q dt^2 + r^2 e^\Phi \delta_{ij} dx^i dx^j \right),$$

where $x^a = (t, x^i)$, and γ_{ab} with r taken to be constant is the induced metric on each hypersurface. This foliation will enable us to consider the “initial value formulation” of

the vacuum Einstein's equation, taking r to be the evolution parameter¹. The appropriate initial data could be chosen as a hypersurface Σ_c located at $r = r_c$, together with its first and second fundamental forms. The first fundamental form is the projection tensor

$$\mathcal{P}_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu,$$

where

$$n_\mu = \frac{1}{\sqrt{AQ}}(dr)_\mu = \left(0, \frac{1}{\sqrt{AQ}}, 0, 0\right),$$

is the unit normal covector. This first fundamental form is closely related to the induced metric, in our coordinate, $\mathcal{P}_{ab} = \gamma_{ab}$, $\mathcal{P}_{\mu r} = \mathcal{P}_{r\mu} = 0$. The second fundamental form is the extrinsic curvature of the hypersurface

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_n \mathcal{P}_{\mu\nu}, \quad (4)$$

also in our choice of the coordinate system we have $K_{\mu r} = K_{r\mu} = 0$. After foliating the background manifold, tensors of type $(0, 2)$ could be decomposed into the following form

$$S_{\mu\nu} = S_{\rho\sigma}\mathcal{P}^\rho{}_\mu\mathcal{P}^\sigma{}_\nu + n_\mu(S_{\rho\sigma}\mathcal{P}^\rho{}_\nu n^\sigma) + n_\nu(S_{\rho\sigma}\mathcal{P}^\rho{}_\mu n^\sigma) + n_\mu n_\nu(S_{\rho\sigma}n^\rho n^\sigma),$$

so the equivalent form of the vacuum Einstein's equation is

$$\begin{aligned} G_{\mu\nu}n^\mu n^\nu &= 0, \\ G_{\mu\nu}\mathcal{P}^\mu{}_\rho n^\nu &= 0, \\ G_{\mu\nu}\mathcal{P}^\mu{}_\rho\mathcal{P}^\nu{}_\sigma &= 0. \end{aligned} \quad (5)$$

The first two lines in (5) are the constraint equations of the initial data $(\mathcal{P}_{\mu\nu}, K_{\mu\nu})$ on the initial hypersurface, and the third line is the evolution equation. According to the Gauss-Codazzi equations the constraint equations could be cast in the following form

$$\hat{R} + K^{ab}K_{ab} - K^2 = 0, \quad (6a)$$

$$D_a(K^a{}_b - \gamma^a{}_b K) = 0, \quad (6b)$$

where \hat{R} is the Ricci scalar of Σ_c , D_a is the covariant derivative compatible with γ_{ab} . The equations (6) are often referred to as the Hamiltonian and momentum constraints, respectively. Equivalently we could choose (γ_{ab}, T_{ab}) as the initial data, here $T_{ab} = \gamma_{ab}K - K_{ab}$ is the Brown-York stress energy tensor. Then the constraint equations can be reformulated as

$$\mathcal{H} = \hat{R} + T^a{}_b T^b{}_a - \frac{T^2}{2} = 0, \quad (7a)$$

$$\mathcal{P}_b = D_a T^a{}_b = 0. \quad (7b)$$

Next we turn to the evolution equations $G_{\mu\nu}\mathcal{P}^\mu{}_\rho\mathcal{P}^\nu{}_\sigma = 0$. Rather than list their concrete forms expressed in terms of (γ_{ab}, T_{ab}) , let us directly come to the following

¹In spite of the fact that r is actually a spacelike coordinate.

conclusion, i.e. *if we perturb the initial data which generates the background spacetime, and demand that it evolves no singularity in the bulk, then the geometry of the perturbed spacetime should be of Petrov type I*. So there are additional constraints of the initial data

$$l^\mu(m_i)^\nu l^\sigma(m_j)^\rho C_{\mu\nu\rho\sigma}|_{\Sigma_c} = 0, \quad (8)$$

where $C_{\mu\nu\rho\sigma}$ is the bulk Weyl tensor, and

$$\begin{aligned} l^\mu &= \frac{1}{\sqrt{2}} \left(\frac{\sqrt{A_c}}{\sqrt{Q_c}} (\partial_t)^\mu - n^\mu \right), \\ k^\mu &= -\frac{1}{\sqrt{2}} \left(\frac{\sqrt{A_c}}{\sqrt{Q_c}} (\partial_t)^\mu + n^\mu \right), \\ (m_i)^\mu &= r^{-1} e^{-\frac{1}{2}\Phi} \sqrt{A_c} (\partial_i)^\mu, \end{aligned} \quad (9)$$

are a set of Newman-Penrose basis vector fields located at the initial hypersurface, here $Q_c = Q(r_c)$, $A_c = A(r_c, x^i)$. Inserting (9) into (8), we get

$$\frac{A_c}{Q_c} C_{titj} + \frac{\sqrt{A_c}}{\sqrt{Q_c}} C_{tij(n)} + \frac{\sqrt{A_c}}{\sqrt{Q_c}} C_{tji(n)} + C_{i(n)j(n)} = 0, \quad (10)$$

and expressing these projections of the bulk Weyl tensor by (γ_{ab}, T_{ab}) , the additional constraint equations will finally become

$$\begin{aligned} \mathcal{E}_{ij} &= 2 \frac{Q_c}{A_c} T^t{}_i T^t{}_j + \frac{T^2}{4} \gamma_{ij} - (T^t{}_t - 2 \frac{\sqrt{A_c}}{\sqrt{Q_c}} D_t) \left(\frac{T}{2} \gamma_{ij} - T_{ij} \right) \\ &\quad - 2 \frac{\sqrt{Q_c}}{\sqrt{A_c}} D_{(i} T^t{}_{j)} - T_{ik} T^k{}_j - \hat{R}^t{}_{itj} - \hat{R}_{ij} = 0, \end{aligned} \quad (11)$$

where $\hat{R}^a{}_{bcd}$, \hat{R}_{ab} represent the Riemann and Ricci tensors of Σ_c . Up till now we have derived all the constraint equations in our initial value formulation, and in the following sections we will see that, on highly accelerated hypersurface these equations give rise to the Navier-Stokes equation.

4 Non-relativistic hydrodynamic expansion and constrained fluctuations

For the background initial data, $\gamma_{ab}^{(B)}$ can be read directly from the line element of initial hypersurface Σ_c , so we can obtain the the background Brown-York tensor $T_{ab}^{(B)}$, and the only nonzero components are

$$\begin{aligned} T^t{}_t^{(B)} &= \frac{2\sqrt{Q_c}}{r_c}, & T^i{}_j^{(B)} &= \left(\frac{\sqrt{Q_c}}{r_c} - \alpha Z \sqrt{Q_c} + \frac{Q'_c \sqrt{A_c}}{2\sqrt{Q_c}} \right) \delta^i{}_j, \\ T^t{}_i^{(B)} &= 0, & T^{(B)} &= \frac{4\sqrt{Q_c}}{r_c} - 2\alpha Z \sqrt{Q_c} + \frac{Q'_c \sqrt{A_c}}{\sqrt{Q_c}}, \end{aligned} \quad (12)$$

here we have used the notations $Q'_c = Q'(r)|_{r=r_c}$, $A'_c(x^i) = \partial_r A(r, x^i)|_{r=r_c}$ for short, and in the rest of this paper the notations Q'_h, Q''_h, A'_h, A''_h will be similar to Q'_c, Q''_c, A'_c, A''_c with r_c replaced by r_h , which represents the radial position of the black hole event horizon. Before imposing perturbation to the background metric, we first consider the non-relativistic limit which will be essential when constructing the non-relativistic hydrodynamics. This could be achieved mathematically by rescaling the t coordinate, $t \rightarrow \frac{t}{\lambda\sqrt{Q_c}}$, then the back ground metric becomes

$$\gamma_{ab}dx^a dx^b = \frac{1}{A_c} \left(-\frac{1}{\lambda^2} (dt)^2 + r_c^2 e^\Phi \delta_{ij} dx^i dx^j \right). \quad (13)$$

The reciprocal of the rescaling parameter λ can viewed as the speed of light and $\lambda \rightarrow 0$ corresponds to the non-relativistic limit. By explicit calculations we find that the Brown-York tensor $T_b^{a(B)}$ and the constraint equations (7) are kept invariant under this rescaling, whilst some coefficients in the additional constraint (11) is changed:

$$\begin{aligned} \mathcal{C}_{ij} \rightarrow \mathcal{C}_{ij} = & \frac{2}{\lambda^2} \frac{1}{A_c} T^t_i T^t_j + \frac{T^2}{4} \gamma_{ij} - (T^t_t - 2\lambda\sqrt{A_c}D_t) \left(\frac{T}{2} \gamma_{ij} - T_{ij} \right) \\ & - \frac{2}{\lambda} \frac{1}{\sqrt{A_c}} D_{(i} T^t_{j)} - T_{ik} T^k_j - \hat{R}^t_{itj} - \hat{R}_{ij} = 0. \end{aligned} \quad (14)$$

Now we take into account the hydrodynamic limit. As is proven in [22], there is a mathematical equivalence between the near horizon limit and the hydrodynamic limit. So let us take a particular initial hypersurface which is close to the black hole horizon at $r = r_h = 2m$. This can be realized by introducing a small parameter ϵ via $r_c - r_h = \epsilon^2$. We would like to link the two small parameters ϵ and λ via $\epsilon = \chi\lambda$, where the constant χ is employed to balance the dimensionality. This identification makes the non-relativistic limit and the hydrodynamic limit occur simultaneously by taking $\lambda \rightarrow 0$. In this double limit, Q_c , $\sqrt{A_c}$ and therefore $T_b^{a(B)}$ could be expanded as

$$\begin{aligned} Q_c &= (\chi\lambda)^2 Q'_h + \frac{1}{2} (\chi\lambda)^4 Q''_h + \dots, \\ \sqrt{A_c} &= \sqrt{A_h} + (\chi\lambda)^2 \alpha Z + \dots, \end{aligned} \quad (15)$$

$$\begin{aligned} T^{t(B)}_t &= 2\chi\lambda \frac{\sqrt{Q'_h}}{r_h} + \dots, & T^{t(B)}_i &= 0 + \dots, \\ T^{i(B)}_j &= \left(\frac{1}{2} \frac{\sqrt{A_h Q'_h}}{\chi\lambda} + \chi\lambda \left(\frac{\sqrt{Q'_h}}{r_h} - \frac{1}{2} \alpha Z \sqrt{Q'_h} + \frac{1}{2} \frac{Q''_h \sqrt{A_h}}{\sqrt{Q'_h}} \right) \right) \delta^i_j + \dots, \\ T^{(B)} &= \left(\frac{\sqrt{A_h Q'_h}}{\chi\lambda} + \chi\lambda \left(4 \frac{\sqrt{Q'_h}}{r_h} - \alpha Z \sqrt{Q'_h} + \frac{Q''_h \sqrt{A_h}}{\sqrt{Q'_h}} \right) \right) + \dots. \end{aligned} \quad (16)$$

Now let us discuss the perturbation theory in the initial value formulation. The most general equations for the fluctuations around the background initial data can be very

complicated, so we restrict ourselves to the following non-relativistic hydrodynamic λ -expansion.

$$\gamma_{ab} = \gamma_{ab}^{(B)} + \sum_{n=1}^{\infty} \gamma_{ab}^{(n)} \lambda^n, \quad (17a)$$

$$T^a{}_b = T^a{}_b^{(B)} + \sum_{n=1}^{\infty} \lambda^n T^a{}_b^{(n)}, \quad (17b)$$

where $(\gamma_{ab}^{(n)}, T^a{}_b^{(n)})$ represent the fluctuation modes. In the above double expansion we will derive the constraint equations for the fluctuations, but before that we need to list some necessary materials in the derivation, i.e. the expansions of the Christoffel symbol, the Riemann tensor \hat{R}_{abcd} and of the Ricci tensor \hat{R}_{ab} , subjecting to the perturbed metric γ_{ab} :

$$\hat{\Gamma}^a{}_{ab} = \hat{\Gamma}^a{}_{ab}^{(B)} + \sum_{n=1}^{\infty} \lambda^n \hat{\Gamma}^a{}_{ab}^{(n)}, \quad (18a)$$

$$\hat{R}^a{}_{bcd} = \hat{R}^a{}_{bcd}^{(B)} + \sum_{n=1}^{\infty} \lambda^n \hat{R}^a{}_{bcd}^{(n)}, \quad (18b)$$

$$\hat{R}_{ab} = \hat{R}_{ab}^{(B)} + \sum_{n=1}^{\infty} \lambda^n \hat{R}_{ab}^{(n)}. \quad (18c)$$

Here the superscript (B) denotes the geometric quantity pertaining to the background metric. The nonzero components of the background Christoffel symbol are

$$\begin{aligned} \hat{\Gamma}_{ti}^{t(B)} &= \frac{\alpha r_c x_i}{\sqrt{A_c}} e^\Phi = \frac{\alpha r_h x_i}{\sqrt{A_h}} e^\Phi + \mathcal{O}(\lambda^1), \\ \hat{\Gamma}_{tt}^{i(B)} &= \frac{1}{\lambda^2} \frac{\alpha x_i}{r_c \sqrt{A_c}} = \frac{1}{\lambda^2} \frac{\alpha x_i}{r_h \sqrt{A_h}} + \mathcal{O}(\lambda^{-1}), \\ \hat{\Gamma}_{ij}^{k(B)} &= \frac{1}{2} (\delta^k{}_i \partial_j \Phi + \delta^k{}_j \partial_i \Phi - \delta^{kl} \delta_{ij} \partial_l \Phi) + \frac{\alpha r_c}{\sqrt{A_c}} e^\Phi (\delta^k{}_i x_j + \delta^k{}_j x_i - \delta^{kl} \delta_{ij} x_l) \\ &= \frac{1}{2} (\delta^k{}_i \partial_j \Phi + \delta^k{}_j \partial_i \Phi - \delta^{kl} \delta_{ij} \partial_l \Phi) + \frac{\alpha r_h}{\sqrt{A_h}} e^\Phi (\delta^k{}_i x_j + \delta^k{}_j x_i - \delta^{kl} \delta_{ij} x_l) + \mathcal{O}(\lambda^1), \end{aligned}$$

where the λ dependences arise from the near-horizon expansion. Since the only necessary expression is $\hat{R}_{itj} + \hat{R}_{ij}$, so we will not list the nonzero components for $\hat{R}^a{}_{bcd}$ and \hat{R}_{ab} separately, rather, by explicit calculations we find

$$\hat{R}_{itj}^{t(B)} + \hat{R}_{ij}^{(B)} = \left(\frac{Q_c}{r_c^2} - \frac{2\alpha Z Q_c}{r_c} + \frac{Q'_c \sqrt{A_c}}{r_c} \right) \gamma_{ij}^{(B)}. \quad (19)$$

Thanks to the above relations, we could finally turn ourselves to the constraint equations for the fluctuations.

(a) Perturbed Hamiltonian constraint

The middle term in the Hamiltonian constraint (7a) can be further decomposed according to the spacial slicing, i.e.

$$\mathcal{H} = \hat{R} + 2T^t_i T^i_t + T^j_i T^i_j - \frac{T^2}{2} = 0. \quad (20)$$

According to (17b) and (19), we can get

$$\begin{aligned} \hat{R} &= 2 \frac{Q'_h \sqrt{A_h}}{r_h} + \mathcal{O}(\lambda^1), \\ T^t_i T^i_t &= -\frac{\gamma^{ij(0)}}{A_h} T^{t(1)}_i T^{t(1)}_j + \mathcal{O}(\lambda^1), \\ \frac{T^2}{2} &= \frac{1}{2} \frac{Q'_h A_h}{(\chi\lambda)^2} + 4 \frac{Q'_h \sqrt{A_h}}{r_h} - \alpha Z Q'_h \sqrt{A_h} + Q''_h A_h + \frac{\sqrt{Q'_h A_h}}{\chi} T^{(1)} + \mathcal{O}(\lambda^1), \\ T^i_j T^j_i &= \frac{1}{2} \frac{Q'_h A_h}{(\chi\lambda)^2} + 2 \frac{Q'_h \sqrt{A_h}}{r_h} - \alpha Z Q'_h \sqrt{A_h} + Q''_h A_h + \frac{\sqrt{Q'_h A_h}}{\chi} T^{i(1)}_i + \mathcal{O}(\lambda^1), \end{aligned} \quad (21)$$

where $\gamma^{ik(0)} = A_h r_h^{-2} e^{-\Phi} \delta^{ik}$, so, at the first nontrivial order of the perturbed Hamiltonian constraint, we have

$$\mathcal{H}^{(0)} = 0 \quad \implies \quad T^{t(1)}_t = -2 \frac{\chi}{\sqrt{Q'_h A_h^3}} \gamma^{ij(0)} T^{t(1)}_i T^{t(1)}_j. \quad (22)$$

Before moving on to the other constraint equations, we need to point out that the derivative operator D_a associated with metric γ_{ab} will receive perturbative corrections, so if our discussion is restricted to the intrinsically curved hypersurface Σ_c , then the covariant form of the momentum and additional constraint will not be strictly expanded as a power series in λ , but as what will be seen, the first nontrivial order of these pseudo expansions still give rise to the continuity equation and Cauchy momentum equation.

(b) Pseudo expansions for momentum and additional constraint

Inserting (17b) into (7b) we get

$$\begin{aligned} D_a T^a_b &= D_a T^{a(B)}_b + \lambda D_a T^{a(1)}_b + \dots, \\ \implies \begin{cases} t\text{-component :} & D_a T^a_t = -\frac{1}{\lambda} \frac{\gamma^{ij(0)}}{A_h} D_i T^{t(1)}_j + \dots, \\ i\text{-component :} & D_a T^{a(1)}_i = \lambda \left(D_t T^{t(1)}_i + D_j T^{j(1)}_i - \chi \frac{\alpha x_i \sqrt{Q'_h}}{2\sqrt{A_h}} e^\Phi \right) + \dots \end{cases} \end{aligned} \quad (23a) \quad (23b)$$

so at the first nontrivial order of the momentum constraints, we get

$$\mathcal{P}_t^{(-1)}(D) = 0 : \quad \gamma^{ij(0)} D_i T^{t(1)}_j = 0, \quad (24a)$$

$$\mathcal{P}_i^{(1)}(D) = 0 : \quad D_t T^{t(1)}_i + D_j T^{j(1)}_i - \chi \frac{\alpha x_i \sqrt{Q'_h}}{2\sqrt{A_h}} e^\Phi = 0. \quad (24b)$$

Due to (17b) and (19) we have

$$\begin{aligned}\frac{T^2}{4} &= \frac{1}{4} \frac{Q'_h A_h}{(\chi\lambda)^2} + 2 \frac{Q'_h \sqrt{A_h}}{r_h} - \frac{1}{2} \alpha Z Q'_h \sqrt{A_h} + \frac{1}{2} Q''_h A_h + \frac{\sqrt{Q'_h A_h}}{\chi} \frac{T^{(1)}}{2} + \mathcal{O}(\lambda^1), \\ T^i{}_k T^k{}_j &= \left(\frac{1}{4} \frac{Q'_h A_h}{(\chi\lambda)^2} + \frac{Q'_h \sqrt{A_h}}{r_h} - \frac{1}{2} \alpha Z Q'_h \sqrt{A_h} + \frac{1}{2} Q''_h A_h \right) \delta^i{}_j + \frac{\sqrt{Q'_h A_h}}{\chi} T^{i(1)}{}_j + \mathcal{O}(\lambda^1), \\ \frac{t}{2} \delta^i{}_j - t^i{}_j &= \lambda \left(\frac{T^{(1)}}{2} \delta^i{}_j - T^{i(1)}{}_j + \chi \frac{\sqrt{Q'_h}}{r_h} \right) + \mathcal{O}(\lambda^1),\end{aligned}$$

so, the first nontrivial order of the additional constraint (14) will be $\mathcal{O}(\lambda^0)$, and we have

$$\mathcal{C}^{i(0)}_j(D) = 0 : T^{i(1)}_j = \frac{\chi}{\sqrt{Q'_h A_h}} \cdot 2\gamma^{ik(0)} \left(\frac{1}{A_h} T^{t(1)}_k T^{t(1)}_j - \frac{1}{\sqrt{A_h}} D_{(k} T^{t(1)}_{j)} \right) + \frac{T^{(1)}}{2} \delta^i{}_j. \quad (25)$$

Note that at the first nonvanishing order of the pseudo expansion, the trace of the additional constraint is precisely the same as the Hamiltonian constraint, i.e.

$$\mathcal{C}^{j(0)}_j(D) = 0 \Leftrightarrow \mathcal{H}^{(0)} = 0.$$

(b') Strictly expanded constraint equations

Alternatively we could express (24) and (25) by ordinary derivative, doing so the momentum and additional constraint will be strictly expanded, then at their first nontrivial order, we get

$$\mathcal{P}_t^{(-1)}(\partial) = 0 : \gamma^{ij(0)}(\partial_i + 3 \frac{\alpha r_h x_i}{\sqrt{A_h}} e^\Phi) T^\tau_j = 0, \quad (26a)$$

$$\begin{aligned}\mathcal{P}_i^{(1)}(\partial) = 0 : \partial_t T^{t(1)}_i + \left(\partial_j + \partial_j \Phi + 3 \frac{\alpha r_h e^\Phi}{\sqrt{A_h}} x_j \right) T^{j(1)}_i \\ - \frac{1}{2} T^{j(1)}_j \partial_i \Phi - \frac{\alpha r_h e^\Phi}{\sqrt{A_h}} x_i T^{(1)} - \chi \frac{\alpha x_i \sqrt{Q'_h}}{2\sqrt{A_h}} e^\Phi = 0,\end{aligned} \quad (26b)$$

and for the additional constraint, we have

$$\mathcal{C}^{i(0)}_j(\partial) = 0 : T^{i(1)}_j = \frac{\chi}{\sqrt{Q'_h A_h}} \cdot 2\gamma^{ik(0)} \left(\frac{1}{A_h} T^{t(1)}_k T^{t(1)}_j - \frac{1}{\sqrt{A_h}} \zeta_{kj} \right) + \frac{T^{(1)}}{2} \delta^i{}_j, \quad (27)$$

where we have used the following short-hand notation

$$\begin{aligned}\zeta_{kj} &= \partial_{(k} T^{t(1)}_{j)} - \partial_{(k} \Phi T^{t(1)}_{j)} + \frac{1}{2} \delta_{kj} \delta^{lm} \partial_l \Phi T^{t(1)}_m \\ &\quad - \frac{\alpha r_h}{\sqrt{A_h}} e^\Phi (x_k T^{t(1)}_j + x_j T^{t(1)}_k - \delta_{kj} \delta^{lm} x_l T^{t(1)}_m).\end{aligned} \quad (28)$$

Interestingly, from (26a) and (27) we find that

$$\mathcal{C}_j^{j(0)}(\partial) = 0 \Rightarrow T_t^{t(1)} = -2 \frac{\chi}{\sqrt{Q'_h}} \frac{\gamma^{ij(0)}}{A_h} \left(\frac{1}{\sqrt{A_h}} T_i^{t(1)} T_j^{t(1)} + 3 \frac{\alpha r_h e^\Phi}{\sqrt{A_h}} x_i T_j^{t(1)} \right), \quad (29)$$

then compare (29) with (22) we will come to the following relation

$$\alpha \cdot \delta^{ij} x_i T_j^{t(1)} = 0. \quad (30)$$

From eqs.(22), (26) and (27) we could also establish the continuity and Cauchy momentum equation, but at the cost of the strict expansion, these equations are not expressed in a covariant form, so we would rather interpret that the final fluid lives in Newtonian spacetime, further the equation (30) implies that there is a vortex in the fluid system.

5 Fluid with inertial force in curved spacetime and fluid with vortex in flat space

In this section we will study the hydrodynamic equations constructed from the constraints of the fluctuations derived in the last section. For equation (24) and (25), we need to introduce the following notations:

$$\tilde{\rho} = A_h^{3/2}, \quad \tilde{\mu} = \sqrt{A_h}, \quad \tilde{\nu} = \frac{\tilde{\mu}}{\tilde{\rho}} = \frac{1}{A_h}, \quad (31)$$

and

$$T_i^{t(1)} = \frac{1}{2} \tilde{\rho} \tilde{v}_i, \quad T^{(1)} = \tilde{p}, \quad (32)$$

we will see that it is suitable to interpret $\tilde{\rho}, \tilde{p}, \tilde{v}_i$ as the density, pressure and velocity field of the dual fluid in curved spacetime, while $\tilde{\mu}, \tilde{\nu}$ as the dynamic and kinematic viscosity.

With the above notations, we can rewrite eq.(24a) in the form

$$D^i(\tilde{\rho} \tilde{v}_i) = 0 \quad (33)$$

which should be considered as the continuity equation for fluid with a stationary density distribution. Then inserting (25) into (24b) with the constant χ chosen as $\sqrt{Q'_h}$, we get

$$\tilde{\rho}(D_t \tilde{v}_i + \tilde{v}^j D_j \tilde{v}_i) = -D_i \tilde{p} + D^j \tilde{d}_{ij} + \tilde{f}_i, \quad (34)$$

where

$$\tilde{d}_{ij} = \tilde{\mu}(D_j \tilde{v}_i + D_i \tilde{v}_j - \gamma_{ij}^{(0)} D^k \tilde{v}_k),$$

$$\begin{aligned}\tilde{f}_i = & -3\alpha \left[2r_h e^\Phi x^j D_{[j} \tilde{v}_{i]} + \frac{1}{\tilde{\mu}} (3\alpha r_h^2 e^{2\Phi} x^j v_j - \frac{1}{3} Q'_h e^\Phi) x_i \right. \\ & \left. + \left(2r_h e^\Phi + \frac{2F(Z)}{\tilde{\nu} r_h} + \frac{3}{\tilde{\mu}} \alpha r_h^2 e^{2\Phi} x^j x_j \right) \tilde{v}_i \right].\end{aligned}\quad (35)$$

Eq. (34) looks much like the Cauchy momentum equation, so we will interpret it as the equation that describes the fluid on the curved hypersurface Σ_c , and so \tilde{d}_{ij} is interpreted as the deviatoric stress for the fluid, and \tilde{f}_i represents a body force. Obviously the acceleration of the black hole is responsible for the appearance of this body force, which vanishes when $\alpha = 0$. In this sense the body force f_i should be thought of as a pure inertial force.

We turn our attention now to the strictly expanded constraint equations with $\alpha \neq 0$, because the case $\alpha = 0$ corresponds to fluctuations around Schwarzschild black hole solution which is somewhat well understood. Since $\alpha \neq 0$, eq.(30) yields

$$\delta^{ij} x_i T_j^{t(1)} = 0, \quad (36)$$

which turns out to be a critical new condition for this case. First of all, the condition (36) implies that eq.(26a) can be rearranged into

$$\partial^j \left(A_h^{-\frac{1}{2}} T_j^{t(1)} \right) = 0. \quad (37)$$

To analyze the “spatial” components of the perturbed momentum constraint, we need to insert (27) and (29) into (26b). The computation is rather complicated, but at the first nontrivial order the equation can be simplified into the following form

$$\begin{aligned}\partial_t T_i^{t(1)} + \frac{1}{2} \partial_i T^{(1)} + \frac{\delta^{jk}}{r_h^2 e^\Phi} \left[\frac{2}{\sqrt{A_h}} T_k^{t(1)} \partial_j T_i^{t(1)} - \partial_j \partial_k T_i^{t(1)} - \frac{2}{\sqrt{A_h}} T_j^{t(1)} T_k^{t(1)} \partial_i \Phi \right. \\ \left. + \partial_j \Phi (\partial_k T_i^{t(1)} - \partial_i T_k^{t(1)}) + T_i^{t(1)} \partial_j \partial_k \Phi - \frac{\alpha r_h e^\Phi}{\sqrt{A_h}} (2x_j \partial_i T_k^{t(1)} + x_j \partial_k T_i^{t(1)} - 2\delta_{jk} T_i^{t(1)} \right. \\ \left. - 5x_j \partial_k \Phi T_i^{t(1)} - 3\delta_{ij} T_k^{t(1)} - 8 \frac{\alpha r_h e^\Phi}{\sqrt{A_h}} x_j x_k T_i^{t(1)}) \right] + \frac{\alpha r_h e^\Phi}{2\sqrt{A_h}} \left(T^{(1)} - \frac{Q'_h}{r_h} \right) x_i = 0,\end{aligned}\quad (38)$$

still we have chosen $\chi = \sqrt{Q'_h}$. Next we are going to interpret eqs. (37) and (38) as the continuity and the Cauchy momentum equations in flat Newtonian spacetime, so in what follows, all the indices will be raised and lowered by δ^{ij} and its inverse δ_{ij} . As the last step we would like to introduce the following “holographic dictionary”

$$\rho = r_h^2 e^\Phi, \quad \mu = 1, \quad \nu = \frac{\mu}{\rho} = \frac{1}{r_h^2 e^\Phi},$$

$$T_i^{t(1)} = \frac{\sqrt{A_h}}{2} \rho v_i, \quad T^{(1)} = \sqrt{A_h} p,$$

where ρ, μ, ν, v_i, p (all without tilde) represent respectively the density, dynamic viscosity, kinematic viscosity, velocity field and the pressure of the fluid living on flat spacetime. Under this dictionary, eq.(37) becomes the continuity equation

$$\partial^j (\rho v_j) = 0, \quad (39)$$

and eq.(38) becomes the standard Cauchy momentum equation

$$\rho(\partial_t v_i + v^j \partial_j v_i) = -\partial_i p + \partial^j d_{ij} + f_i, \quad (40)$$

for the viscous fluid, with the symmetric traceless tensor

$$d_{ij} = \mu \left(\partial_j v_i + \partial_i v_j - \delta_{ij} \partial^k v_k \right), \quad (41)$$

representing the deviatoric stress, and

$$\begin{aligned} f_i = & 2F(Z) \left(x^j \partial_{[j} v_{i]} - v_i + \frac{1}{2} \rho v^2 x_i \right) \\ & + \frac{\alpha \rho}{r_h \sqrt{A_h}} \left[\frac{1}{2} \left(p + \frac{Q'_h}{r_h \sqrt{A_h}} \right) x_i - \left(10 + 6F(Z)w^2 + 9 \frac{\alpha \rho}{r_h \sqrt{A_h}} w^2 \right) v_i \right] \end{aligned} \quad (42)$$

representing a body force. In the last force term, we have used the short-hand notations

$$w = \sqrt{x^i x_i} = \left(\sqrt{\frac{1-Z}{1+Z}} \left(\frac{(1+2\alpha m Z)^2}{1-Z^2} \right)^{\alpha m} \right)^{\frac{1}{1-(2\alpha m)^2}}$$

and

$$F(Z) = \frac{2}{w^2} (3\alpha m Z^2 + Z - \alpha m - 1).$$

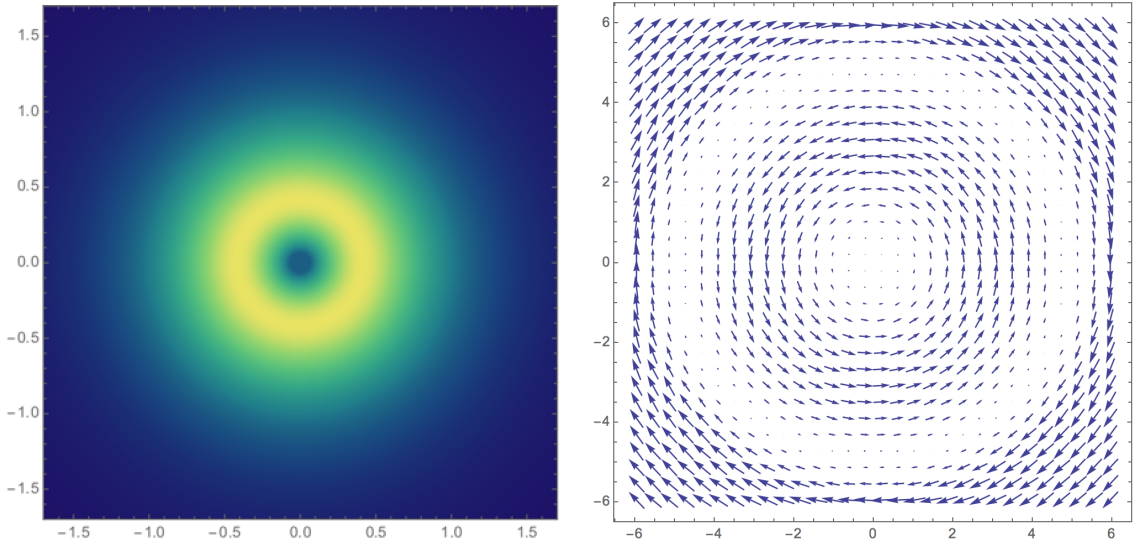


Figure 1: Density (left) and velocity field (right) for the vortex fluid (density increases as the color goes from dark blue to bright yellow)

It is clear that the second line in eq.(42) is a pure inertial force because of the overall factor α . As for the first line in eq.(42), we could recognize the first term as a kind of Coriolis force and the second term as a linear resistance force, so only the last term

on the first line remains a mystery. Nevertheless one can recognize that this last force term is proportional to the kinetic energy density of the fluid component, which also sounds good. Anyway we should not forget the role of (36), which is now rewritten in terms of the holographic dictionary as

$$x^i v_i = 0, \quad (43)$$

i.e. the velocity field of the fluid is always perpendicular to its spacial displacement. This is nothing but a vortex condition. In other words, the fluid we constructed on the flat Newtonian spacetime is precisely a holographic vortex.

To make an intuitive impression of the vortex fluid, we present a plot for the density distribution and the velocity field for the particular choice of parameters $\alpha = r_h = 1$ and $m = 0.5$. These plots are given in Fig.1. These plots fully justify the conclusion that the fluid indeed behave as a vortex. That the velocity field does not vanish in the far zone can be interpreted by the presence of the Coriolis-like force term in the fluid equation – it just reflects the fact that the coordinate frame that we have chosen is not an asymptotically inertial frame. However, even if we change the coordinate frame into an asymptotically inertial one, the vortex behavior should not be altered, because the vortex flow of the inner ring is different from that of the outer part, so the circular flows of both parts cannot be simultaneously removed by a mere change of coordinates.

6 Conclusion

Unlike the ordinary static black holes with maximally symmetric horizons, the C-metric black hole represents two black holes under constant proper acceleration. The acceleration of the black hole squeezes the horizon surface, leaving less symmetries than the non-accelerating black holes. Our construction reveals that Gravity/Fluid correspondence can be realized in terms of Petrov I boundary condition even for black holes with less symmetries than the usual static black holes with maximally symmetric horizons.

To be more concrete, we have realized two fluid systems from the vacuum C-metric black hole solution, one lives on the curved near horizon hypersurface Σ_c , the other lives on a flat Newtonian spacetime. Both fluid systems possess non-constant but stationary density distributions and kinematic viscosities, and so they are compressible viscous fluids subject to extra body forces.

Compared with previous studies on Gravity/Fluid correspondences, the present work differs in two major aspects. The first difference occurs in the curved space fluid equations. To our knowledge, all previous studies on cases with curved near horizon hypersurfaces had led to incompressible Navier-Stokes equations, even if the hypersurface is anisotropic [50] or rotating [51]². However, in the case with C-metric background, the curved space fluid equation is (34), which, together with (33), constitute a system

²On the same day that the first version of this paper appeared on arXiv, another interesting

of *compressible* viscous fluid with stationary density distribution. The second difference lies in that the extra body forces arising from the C-metric black hole case can have more appropriate physical interpretations. For the first (curved space) fluid system, the extra force is purely an inertial force due to the acceleration of the background geometry. For the second (flat space) fluid system, the extra force are consisted of an inertial force term, a Coriolis-like term, a linear resistance term and a term proportional to the kinetic energy density of the fluid. It is remarkable that the combination of all these complicated force terms give rise to a pure vortex behavior for the fluid system in this case.

Before ending, let us stress that the Gravity (with curved horizon)/Flat space fluid correspondence realized in [47, 48] and the present work seems to rely on the conformal flatness of the horizon surface of the background geometry. However, going through the details of the construction, it is evident that such correspondence only requires the existence of a map from the near horizon hypersurface to the flat space, be it conformal or not. Therefore, it is tempting to consider other cases with more complicated, less symmetric black hole backgrounds. Doing so one might be able to get more general fluid systems with less constraints on the density distributions and/or kinematic viscosities. For this purpose, the black ring geometry in 5 dimensions may be a good choice as background geometry. We leave the study of such backgrounds to later works.

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Appendix

We rewrite the C-metric line element as

$$ds^2 = \frac{1}{\mathcal{A}} \left(-Q dt^2 + \frac{dr^2}{Q} + d\tilde{s}^2 \right),$$

where

$$d\tilde{s}^2 = r^2 \left(\frac{1}{P} d\theta^2 + P \sin^2 \theta d\varphi^2 \right).$$

If we consider θ as function of the new coordinate w , then $d\tilde{s}^2$ becomes

$$d\tilde{s}^2 = r^2 \left[\frac{1}{P} \left(\frac{d\theta}{dw} \right)^2 dw^2 + P \frac{\sin^2 \theta}{w^2} w^2 d\varphi^2 \right].$$

paper [52] appeared also on arXiv, which constructed incompressible Navier-Stokes equation with Coriolis force from general rotating black holes. It is interesting to ask whether the fluid system in that paper has a vortex behavior or not.

It is clear that if we take

$$\frac{1}{P} \left(\frac{d\theta}{dw} \right)^2 = P \frac{\sin^2 \theta}{w^2} w^2, \quad (44)$$

then $d\tilde{s}^2$ will become

$$d\tilde{s}^2 = r^2 \frac{1}{P} \left(\frac{d\theta}{dw} \right)^2 (dw^2 + w^2 d\varphi^2) \quad (45)$$

which is a Weyl rescaling of the line element of a flat Euclidean plane in radial coordinate.

Now it remains to solve (44) as a separable ordinary differential equation. It turns out that the function $\theta(w)$ cannot be written out explicitly. Rather, its inverse function, $w(\theta)$, can be written in explicit form,

$$w = \left[\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \left(\frac{1 + 2\alpha m \cos \theta}{\sin \theta} \right)^{2\alpha m} \right]^{\frac{1}{1 - (2\alpha m)^2}}.$$

With this result in hand, we can calculate the Weyl factor in (45), and, in the end, $d\tilde{s}^2$ is turned into

$$d\tilde{s}^2 = r^2 \frac{(1 + 2\alpha m \cos \theta) \sin^2 \theta}{w^2} (dw^2 + w^2 d\varphi^2).$$

Finally, taking the coordinate change

$$x^1 = w(\theta) \cos \varphi, \quad x^2 = w(\theta) \sin \varphi,$$

the C-metric becomes

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{A} \left(-Q dt^2 + \frac{dr^2}{Q} + r^2 e^{\Phi(x^i)} \delta_{ij} dx^i dx^j \right).$$

where

$$A(r, x^i) = ([1 + \alpha r \cos(\theta(x^i))]^2, \\ e^{\Phi(x^i)} = \frac{[1 + 2\alpha m \cos(\theta(x^i))] \sin^2(\theta(x^i))}{w^2}.$$

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